Agricultural Production and Technological Change

Advanced Producer Theory and Analysis: Productivity and Efficiency

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AREC 705: Week 7

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Efficiency and Productivity

Productivity

is the ratio of output to input.

Productivity Growth

is the *difference* between the growth in outputs and the growth in inputs.

Variation in Productivity

is a residual.

> *"... a measure of our ignorance."*
>
> *(Abramovitz 1956)*

What constitutes the residual?

- Production technology
- Scale of operation
- Operating environment
- **Efficiency**

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The comparison between observed and optimal values of output and input.

What constitutes the residual?

- Production technology
- Scale of operation
- Operating environment
- **Efficiency**

The comparison between observed and optimal values of output and input.

1. Which outputs and inputs should be included in the comparison?
    - How do we separate efficiency from other components of the residual?
2. How are multiple outputs and inputs to be weighted?
    - How do we account for differences in prices across time and firms?
    - How do we account for missing market prices (e.g., externalities)?
3. How are the optimal values determined?

Efficiency and Productivity Terminology

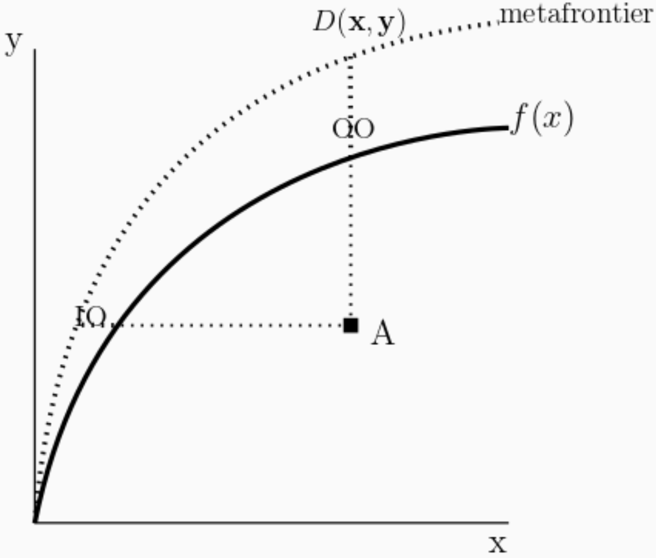
Output-oriented technical efficiency (OO): A plan (combination of inputs and outputs) is *technically inefficient* if the observed input levels can produce a higher level of output.

Input-oriented technical efficiency (IO): A production plan (combination of inputs and outputs) is *technically inefficient* if the observed output level can be produced using fewer inputs.

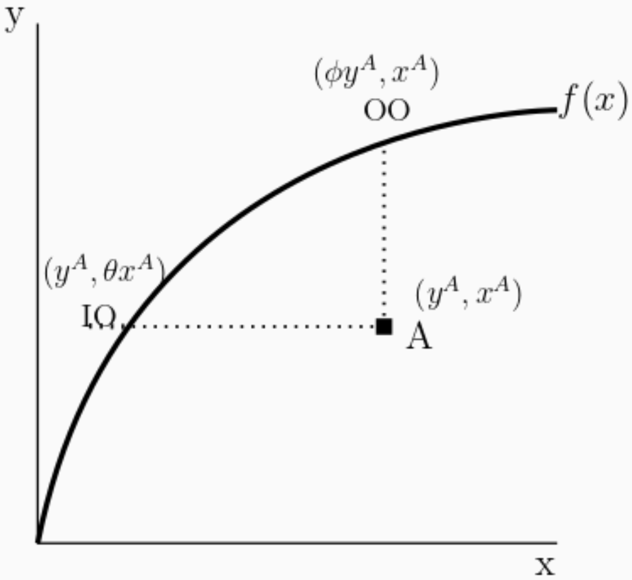
Technical efficiency: An input-output combination may appear inefficient for one technology, but be efficient for a different technology.

Metafrontier analysis: The envelope of all group frontiers, and provides a homogenous boundary for all heterogeneous groups. It thus represents the best-practice technology.

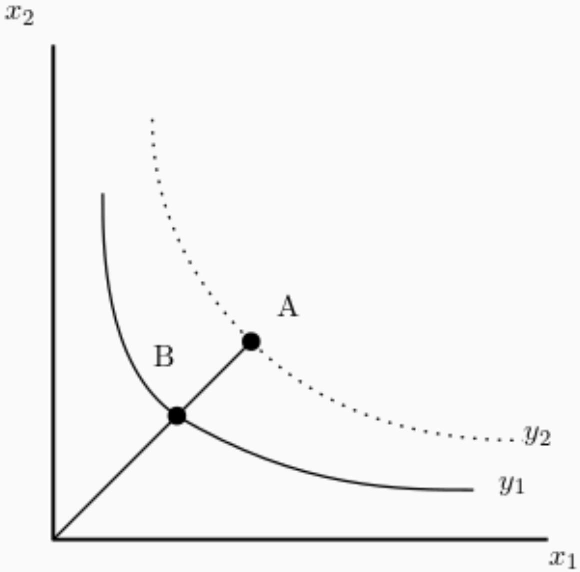
Efficiency and Productivity Terminology



Measuring Efficiency and Inefficiency – Single Input & Output



Measuring Efficiency and Inefficiency – Multiple Input, Single Output

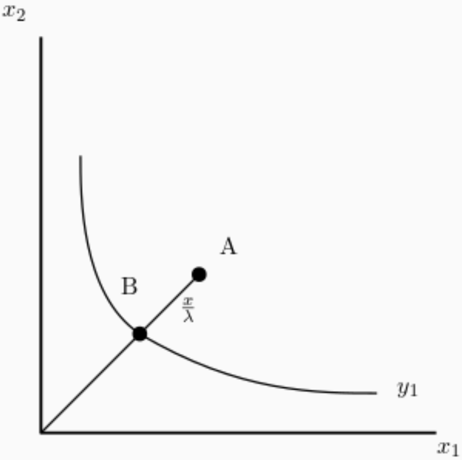


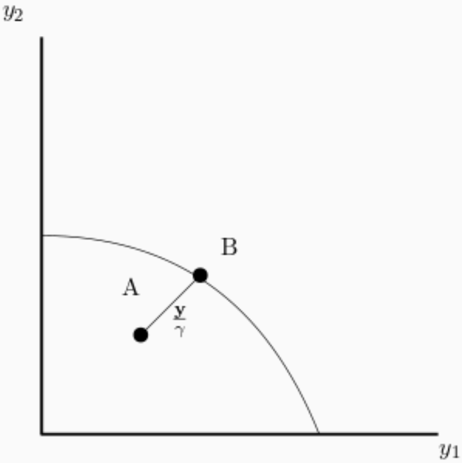
Efficiency and Productivity Terminology

The distance function: The radial distance from an observed production set to the technically feasible set. Distance functions are commonly used in multiple-output production functions when...

- Cannot separate the contribution of inputs to any one output
- Cannot combine outputs into one composite measure

Measuring Efficiency and Inefficiency – Multiple Output





The Distance Function

IO inefficiency can always be measured by the IO distance function (D_{IO}):

An example with one output: (equivalent to production function)

$$D_{IO}(y, \mathbf{x}) = \max_{\lambda}\{\lambda | f(\mathbf{x}/\lambda) \geq y\}$$

With multiple outputs and multiple inputs:

$$D_{IO}(\mathbf{y}, \mathbf{x}) = \max_{\lambda}\{\lambda | f(\mathbf{x}/\lambda) \in L(\mathbf{y})\}$$

Where

$$I(y) = \{x : x \in L(y) \text{ and } \lambda x \notin L(y) \text{ if } 0 \leq \lambda < 1\}$$

are the input isoquants of the technically efficient set.

The Distance Function

Properties of D_{IO}

- decreasing in each output level
- increasing in each input level
- homogeneous of degree 1 in feasible input vector $\mathbf{x}$
- concave in $\mathbf{x}$

The Distance Function

OO inefficiency can always be measured by the OO distance function (D_{OO}):

An example with one output: (equivalent to production function)

$$D_{OO}(y, \mathbf{x}) = \min_{\gamma}\{\gamma | \frac{y}{\gamma} \leq f(\mathbf{x})\}$$

With multiple outputs and multiple inputs:

$$D_{OO}(\mathbf{y}, \mathbf{x}) = \min_{\gamma}\{\gamma | \frac{\mathbf{y}}{\gamma} \leq \mathcal{P}(\mathbf{x})\}$$

Where $\mathcal{P}(\mathbf{x})$ gives the set of feasible output vectors.

The Distance Function

Properties of D_{IO}

- decreasing in each output level
- increasing in each input level
- homogeneous of degree 1 in feasible input vector $\mathbf{x}$
- concave in $\mathbf{x}$

Properties of D_{OO}

- decreasing in each input level
- increasing in each output level
- homogeneous of degree 1 in $\mathbf{y}$
- concave in $\mathbf{y}$

The Distance Function

Finally, note that both input and output distance functions are functions of both x and y. So, formally we define them through the homogeneity restrictions:

$$\frac{D_{IO}}{x_1} = f\left(\frac{x_2}{x_1}, ..., \frac{x_j}{x_1}, \mathbf{y}\right)$$

$$\frac{D_{OO}}{y_1} = f\left(\mathbf{x}, \frac{y_2}{y_1}, ..., \frac{y_m}{y_1}\right)$$

Data Envelopment Analysis (DEA): A nonparametric linear programming method. Random noise is absorbed into frontier estimates because it uses observed input and output sets.

- DEA is largely atheoretical. Main strength is the lack of parameterization. No assumptions about the form of the technology.
- This is a deterministic estimator in that it is determined by the data and not underlying theory.
- Deterministic frontier functions dictate that the deviation of an observation from the theoretical maximum is attributed solely to the inefficiency of the firm.

Some Common DEA Methods

In its simplest form, DEA assumes constant returns to scale (CRS) (Charnes et al. 1978)...

Define (IO) efficiency as a ratio of weighted sum of (k) outputs to weighted sum of (j) inputs: $\frac{\sum_k u_k y_{ki}}{\sum_j v_j x_{ji}}$

Then we can write the maximization problem for observation (or producer) i_0 as:

$$\max \quad \frac{\sum_k u_k y_{ki_0}}{\sum_j v_j x_{ji_0}}$$

$$\text{s.t.} \qquad 1 \geq \frac{\sum_k u_k y_{ki}}{\sum_j v_j x_{ji}}, \quad \forall i = 1, ..., \; n$$

$$u_k, \; v_j \geq \epsilon > 0, \quad k = 1, ..., \; K; \quad j = 1, ..., \; J$$

Some Common DEA Methods

In its simplest form, DEA assumes constant returns to scale (CRS)...

To avoid maximizing a problem with unknowns in the numerator and denominator, we can define $\bar{u_k} = tu_k$ and $\bar{v_j} = tv_j$, where $t = \left(\sum_j v_j x_{ji_0}\right)^{-1}$, and rewrite this as:

$$\max \quad \theta_0 = \sum_k \bar{u_k} y_{ki_0}$$

$$\text{s.t.} \qquad -\sum_j \bar{v_j} x_{ji} + \sum_k \bar{u_k} y_{ki} \leq 0, \quad \forall i = 1, ..., \ n$$

$$\sum_j \bar{v_j} x_{ji_0} = 1$$

$$\bar{u_k}, \ \bar{v_j} \geq \bar{\epsilon}, \quad k = 1, ..., \ K; \quad j = 1, ..., \ J$$

Some Common DEA Methods

For the one-input–one-output case, we have (for producer i):

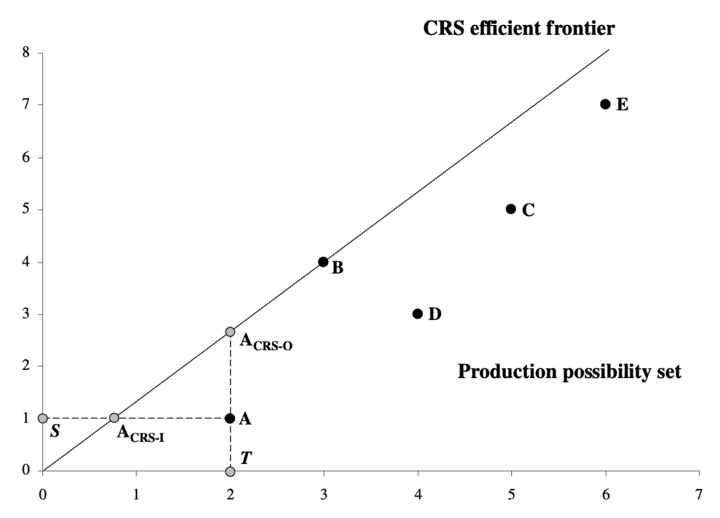
raw (IO) efficiency$_i = \theta_i = \frac{uy_i}{vx_i}$

Since we care about relative efficiency, we can rewrite this as:

$$\begin{aligned}\text{(IO) efficiency}_i &= \frac{\text{raw efficiency}_i}{\max\{\text{raw efficiency}_1, \ldots, \text{raw efficiency}_J\}} \\ &= \frac{\frac{uy_i}{vx_i}}{\max\{\frac{uy_1}{vx_1}, \ldots, \frac{uy_J}{vx_J}\}} = \frac{\frac{y_i}{x_i}}{\max\{\frac{y_1}{x_1}, \ldots, \frac{y_J}{x_J}\}}\end{aligned}$$

Note that to get OO efficiency we simply invert the ratio in the maximization problem, i.e., consider the ratio of the sum of weighted inputs to the sum of weighted outputs.

CRS DEA



CRS efficient frontier
8
7
6
5
4
3
2
1
0
0
1
2
3
4
5
6
7
E
C
B
D
A_CRS-O
Production possibility set
S
A_CRS-I
A
T

Some Common DEA Methods

A slight improvement on the CRS assumption comes from VRS...

This was first modeled in Banker et al. (1984) and involves the additional variable u_0:

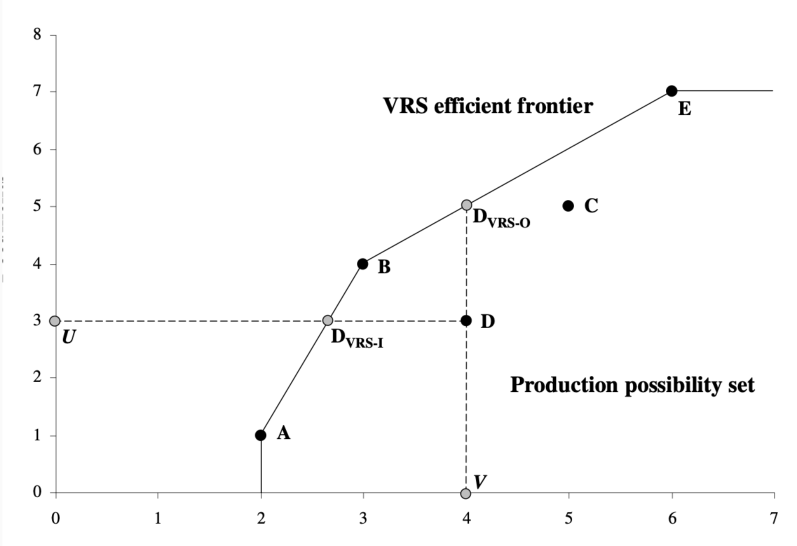
$$\max \quad \sum_k \bar{u}_k y_{ki_0} - \bar{u}_0$$

$$\text{s.t.} \qquad -\sum_j \bar{v}_j x_{ji} - \bar{u}_0 + \sum_k \bar{u}_k y_{ki} \leq 0, \quad \forall i = 1, \ldots, \ n$$

$$\sum_j \bar{v}_j x_{ji_0} = 1$$

$$\bar{u}_k, \ \ \bar{v}_j \geq \bar{\epsilon}, \quad k = 1, \ldots, \ K; \quad j = 1, \ldots, \ J; \bar{u}_0 \text{ unrestricted}$$

VRS DEA



8
7
6
5
4
3
2
1
0
VRS efficient frontier
E
C
D_VRS-O
B
D
U
D_VRS-I
Production possibility set
A
V
0
1
2
3
4
5
6
7

Stochastic frontier analysis (SFA): A parametric approach that asserts a particular functional form for $f(x)$ and uses the data to estimate the parameters.

- SFA is based on theory, need to select a production function and a form of
- SFA has distribution-free and distribution approaches. These relate to the assumptions on the structure of the error term.
- SFA distribution approaches allow you to separate random noise or disturbances from your estimates of efficiency (but this requires assumptions on the distribution of the error term)

Some Common Production Functions for SFA

Cobb-Douglas (without technical inefficiency):

$$y = f(x) = A \prod_{j=1}^{J} x_j^{\beta_j}$$

$$\Rightarrow \ln y = \beta_0 + \sum_j \beta_j \ln x_j$$

Where $\beta_0 = \ln A$

With technical change we can specify Cobb-Douglas as: $\ln y = \beta_0 + \sum_j \beta_j \ln x_j + \beta_t t$

Which implies that the speed of technical change is zero.

Some Common Production Functions for SFA

Cobb-Douglas with OO:

$$y = f(x)e^{-u}$$

$$\ln y = \ln f(x) - u = \beta_0 + \sum_j \beta_j \ln x_j - u$$

Cobb-Douglas with IO:

$$y = f(xe^{-\eta})$$

$$\ln y = \beta_0 + \sum_j \beta_j \ln x_j - \left(\sum_j \beta_j\right)\eta$$

Some Common Production Functions for SFA

Because Cobb-Douglas is homogeneous of degree 1, there is (essentially) no difference between IO and OO technical inefficiency. This implies that none of the economic measures we care about are affected by the presence of inefficiency:

Elasticity of output: $\frac{\partial \ln y}{\partial \ln x_j} = \beta_j$

Returns to scale: $\sum_j \frac{\partial \ln y}{\partial \ln x_j} = \sum_j \beta_j$

Elasticity of substitution: $\sigma_{ij} = 1$

Some Common Production Functions for SFA

Translog (without technical inefficiency):

$$\ln y = \beta_0 \sum_j \beta_j \ln x_j + \frac{1}{2} \sum_j \sum_k \beta_{jk} \ln x_j \ln x_k, \;\; \beta_{jk} = \beta_{kj}$$

$$\Rightarrow \ln y = \beta_0 + \sum_j \beta_j \ln x_j$$

With technical change we can write:

$$\ln y = \beta_0 \sum_j \beta_j \ln x_j + \frac{1}{2} \sum_j \sum_k \beta_{jk} \ln x_j \ln x_k + \beta_t t \frac{1}{2} \beta_{tt} t^2 + \sum_j \beta_j t \ln x_j t$$

Where the rate of technical change is: $\beta_t \beta_{tt} t + \sum_j \beta_{jt} ln x_j$

Some Common Production Functions for SFA

Translog with OO:

$$\ln y = \beta_0 \sum_j \beta_j \ln x_j + \frac{1}{2} \sum_j \sum_k \beta_{jk} \ln x_j \ln x_k - u, \;\; \beta_{jk} = \beta_{kj}$$

Translog with IO:

$$\ln y = \beta_0 \sum_j \beta_j (\ln x_j - \eta) + \frac{1}{2} \sum_j \sum_k \beta_{jk} (\ln x_j - \eta)(\ln x_k - \eta), \;\; \beta_{jk} = \beta_{kj}$$

$$\Rightarrow \ln y = \beta_0 \sum_j \beta_j \ln x_j + \frac{1}{2} \sum_j \sum_k \beta_{jk} \ln x_j \ln x_k - \eta \left[\sum_j \beta_j + \sum_j \left(\sum_k \beta_{jk} \ln x_k \right) \right] + \frac{1}{2} \eta^2 \sum_j \sum_k \beta_{jk}$$

Some Common Production Functions for SFA

Assuming the Translog production function leads to a few differences in economic measures we might be interested in when using IO and OO TE.

Translog with OO technical inefficiency:

Elasticity of output: $\frac{\partial \ln y}{\partial \ln x_j} = \beta_j + \sum_k \beta_{jk} \ln x_k$

Returns to scale: $\sum_j \frac{\partial \ln y}{\partial \ln x_j} = \sum_j (\beta_j + \sum_k \beta_{jk} \ln x_k)$

With IO technical inefficiency:

Elasticity of output: $\frac{\partial \ln y}{\partial \ln x_j} = \beta_j + \sum_k \beta_{jk} \ln x_k + (\sum_k \beta_{jk}) \eta$

Returns to scale: $\sum_j \frac{\partial \ln y}{\partial \ln x_j} = \sum_j (\beta_j + \sum_k \beta_{jk} \ln x_k + (\sum_k \beta_{jk}) \eta)$

Next Time

We will proceed with an example of applying the DEA approach. It might be helpful to bring a computer...

Assigned Reading:

Hailu, A. and Veeman, T.S. (2001), Non-parametric Productivity Analysis with Undesirable Outputs: An Application to the Canadian Pulp and Paper Industry. *American Journal of Agricultural Economics, 83*: 605-616. `https://doi.org/10.1111/0002-9092.00181`